2.7 Artifact: Measurement of UAS Moments of Inertia and Placement of the Center of Gravity-- Philip Ahlers

2.7.1 Abstract
During the design of the UAS, estimations were made for the aircraft's mass and moments of inertia. Most of the aircraft components were estimated as point masses, or constant density three dimensional shapes. This model is also what gave approximate values for the aircraft's mass. Since the moments of intertie of the aircraft effect the aircraft's dynamic modes and responses to disturbances, this experiment determined the true MOI and ensured that handling characteristics were maintained. In order to validate the design, and ensure that the aircraft's CG is within designed requirements, certain measurements and changes to the aircraft were made. In this experiment, the aircraft's mass and moments of intertie were measured to validate the design estimations, and the CG adjusted to ensure that the static margin was as designed. A rig that holds the aircraft was used to measure and set CG locations, and an inertial twist test or bifilar torsion pendulum was used to experimentally determine moments of intertie, and the aircraft's final weight was measured to ensure safety and that the aircraft is within designed limits.

2.7.2 Background
During the design and analysis of the UAS, the static margin was to be in a range that provided Level I handling characteristics based on estimated moments of inertia and mass. In order to ensure the aircraft functions as designed, the aircraft's weight, CG, and moments of inertia were experimentally determined and compared to the designed or planned data. The CG will be placed on the aircraft to give the aircraft the designed static margin.

2.7.3 Procedure
To measure the aircraft's weight, a simple digital scale was used to measure the aircraft's weight within a hundredth of a pound. The aircraft's center of gravity was measured by simply balancing the aircraft using a custom built rig, pictured below. The tail boom was leveled using a bubble level. The mass and CG of the rig used to hold the aircraft was measured and its effect on the rig+plane combination was found.

Figure 2.7.3-1 Aircraft CG Rig

The object of this was to have the ability to measure that balance point to a reference on the aircraft. The zero coordinate was the tip of the aircraft's spinner.
Using this rig, and design data for the aircraft's neutral point, the static margin was set to the designed static margin of 12.5%. We were able to obtain a static margin of 12.5% by adding weight to the tail. This weight and CG was recorded, and then the MOI were gathered using the below technique.
The moments of inertia were experimentally determined using a technique involving a bifilar pendulum. Below is a diagram of the setup of this technique.

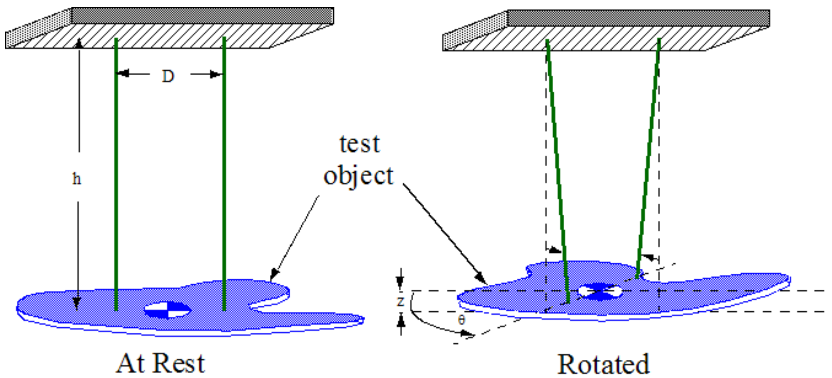

Figure 2.7.3-2 Bifilar Torsional Pendulum (mathworks.com)

The aircraft was hung by two points, each equidistant from the CG. The aircraft is then excited to oscillate around the CG, and the period is timed and recorded. This is done for each of the plane's axis. The will allow the calculation of the mass moments of inertia for each axis. By knowing that the systems total kinetic energy is in this angular oscillation, we can derive equations to estimate mass moments of inertia, by neglecting viscous drag, and losses by stretching of the hanging wire or string. This equation simplifies to:

$$I = \frac{mgD^2}{4h{\omega_n}^2}$$

The lengths D and h are explained in Figure 2. Again, this equation neglects any damping forces that potentially could produce a slightly different damped frequency instead of the true natural frequency. The only measurements needed are the distances from the CG to the string attachment, the length of the strings, and the mass of the system. Then the natural frequency can be found by timing the periods.
To validate the setup and equation, a simple wooden 2x4 was suspended and excited (the equation also requires a small angle). A span of 10 oscillations was timed, and from that an average period was found. The inverse of the period is the natural frequency, and used to calculate the moment of inertia. Since the wooden 2x4 is of constant density and a simple shape, its mass moment of inertia can be easily calculated simply by its dimensions and mass. The value calculated by the dimensions and mass was compared to the value produced by the bifilar pendulum. The error was under 1.5%, and was deemed extremely accurate. This error could have been caused by the 2x4's rounded edges or damping forces from the air or stretch in the suspension system. The same method was used to find the UAS's Izz, Ixx, and Iyy.

Figure 2.7.3-3 Aircraft Suspension

Above in Figure 3, the aircraft is shown suspended in each axis. Great care was taken to ensure the points of suspension were equidistant from the CG of the aircraft. This prevents any precession during the oscillation, and the linearized equation requires this. The suspension strings also must remain parallel, this was checked before each new excitation of the system.

2.7.4 Results

The CG and CG range were part of this experiment, and as such, during the measurement, placement, and verification of the CG location, limits of control power were used to calculate available CG forward and aft limits. Below is a table describing these data points.

Static Margin	Control Power	elevator angle to trim
23.3	1.07	-0.5
18.3	1.15	1
13.3	1.29	1.5
10.9	1.87	2
8.4	2.46	3

Table 1: Static Margin and Control Power Limits

Since the aircraft built was slightly different than the aircraft designed due to manufacturing errors, problems, or complications, the aircraft was re-modeled as constructed and re-evaluated. Above in the table—our design static margin of 12.5% gives the aircraft a control power of 1.36. This is slightly higher than the design control power at that static margin, which used to be 1.2. The aircraft can operate from a static margin of 24% to a static margin of 9.9%. This translates to a CG range that spans 1.33 inches. This is the safe operating range of the CG of this particular aircraft based on computer modeling. Since the complete aircraft had a CG .6 inches forward of the intended location, a combination of reducing wiring weight, skid weight, and the addition of tail weight was used to move the CG aft. The aircraft was set for the 12.5% static margin by adding an empty C02 canister into the rocket motor casing. This holds the canister snugly and it cannot move. These canisters can then be filled or cut to change the desired static margin or CG placement. Below is a table of three weights that were designed and tested to produce three CG/static margins.

Table 2.7.4-1 Static Margin & Weight addition

Static Margin	Weight Added to Tail
22%	40g/1.4oz
13%	60g/2.1oz
10%	70g/2.4oz

Moments of inertia were again, estimated based on component placements and masses during the design phase. These estimations were refined in this artifact and the same MATLAB code used in CDR to ensure the aircraft maintained level 1 handling. Below is a table of the final MOIs found during this experiment, and the deviation or percent difference from the estimations in the design phase. Since the weight of the UAS is around 6.4lbs, which is about a 17% increase from design weight, the increase in Izz, Ixx, and Iyy weren't unexpected.

Table 2.7.4-2: MOI Experimental and Estimated

	FINAL	ESTI	DIFF (%)
IZZ			
kg m^2	0.4049	0.32	
lb in^2	1383.9	1095	0.21
IXX			
kg m^2	0.1644	0.118	
lb in^2	561.7	402.9	0.28
IYY			
kg m^2	.2837	0.208	
lb in^2	967.2	711	.36

2.7.5 Conclusion

Based on the improved data found during this experiment, it has been proven that while our aircraft's construction was completed with a plane that is heavier than estimated, the aircraft is still fully capable of flight. A CG range was calculated and determined; accurate moments of inertia were experimentally determined and used to update the aircraft's handling qualities, and control power remains within designed bounds.

2.7.6 Data

IZZ RIG + PLANE							**IZZ RIG**												
g	d	h	m				g	d	h	m									
9.81	0.163513	0.8604	3.28664				9.81	0.163513	1.495	0.412									
	time for 1	period	top	bot	MOI			time for 1	period	top	bot	MOI							
	80.96	8.096	14.12559	33.96723	0.415859			53.43	5.343	0.771225	59.02023	0.013067							
	81.5	8.15	14.31466	33.96723	0.421425			53.48	5.348	0.772669	59.02023	0.013092					**FINAL**	**EST**	**DIFF (%)**
	81.06	8.106	14.16051	33.96723	0.416887			53.4	5.34	0.770359	59.02023	0.013052							
avg:	81.17333	8.117333	14.20025	33.96723	0.418057		avg:	53.43667	5.343667	0.771418	59.02023	0.01307		**IZZ**		kg m^2	0.404987		
																lb in^2	1383.9	1095	0.208758
IXX RIG + PLANE							**IXX RIG**												
g	d	h	m				g	d	h	m									
9.81	0.19685	0.8382	3.28664				9.81	0.1968	1.076	0.48									
	time for 1	period	top	bot	MOI			time for 1	period	top	bot	MOI							
	42.34	4.234	5.5993	33.09081	0.16921			18.09	1.809	0.149203	42.47878	0.003512							
	42.02	4.202	5.514982	33.09081	0.166662			18.35	1.835	0.153523	42.47878	0.003614							
	42.19	4.219	5.559696	33.09081	0.168013			17.83	1.783	0.144945	42.47878	0.003412							
avg:	42.18333	4.218333	5.557939	33.09081	0.167962		avg:	18.09	1.809	0.149203	42.47878	0.003513		**IXX**		kg m^2	0.164449		
																lb in^2	561.7	402.9	0.282713
IYY RIG + PLANE							**IYY RIG**												
g	d	h	m				g	d	h	m									
9.81	0.3175	0.9874	3.28664				9.81	0.2921	0.99	0.512									
	time for 1	period	top	bot	MOI			time for 1	period	top	bot	MOI							
	37.63	3.763	11.50581	38.98099	0.295165			21.25	2.125	0.483793	39.08363	0.012378							
	37.64	3.764	11.51192	38.98099	0.295321			21.09	2.109	0.476535	39.08363	0.012193							
	37.81	3.781	11.61614	38.98099	0.297995			21.45	2.145	0.492943	39.08363	0.012613							
avg:	37.69333	3.769333	11.54457	38.98099	0.29616		avg:	21.26333	2.126333	0.484401	39.08363	0.012395		**IYY**		kg m^2	0.283766		
																lb in^2	967.2	711	0.360338

Figure 2.7.6-1 MOI Raw Data

2.7.6 References

http://www.mathworks.com/